L'Hopital & Parts Review #1

1. $$\lim_{x\to 0} \frac{2x^3-3x^2+5x}{x^3-x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{6x^2-6x+5}{3x^2-1} = -5$$

2. $$\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}} = \frac{0}{0}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{-\sin x}{1} = -1$$

3. $$\lim_{x\to \infty} \frac{\ln x}{2^x} = \frac{\infty}{\infty}$$

$$\lim_{x\to \infty} \frac{\frac{1}{x}}{2^x \cdot \ln 2} = 0$$

4. $$\lim_{x\to 0} \frac{\tan x - \sin x}{x^2 \tan x} = \frac{0-0}{0}$$

$$\lim_{x\to 0} \frac{\sec^2 x - \cos x}{2x\tan x + x^2\sec^2 x} = \frac{1-1}{0+0}$$

$$\lim_{x\to 0} \frac{2(\sec x)\sec x\tan x + \sin x}{2\tan x + 2x\sec^2 x + 2x\sec^2 x + x^2\, 2(\sec x)\sec x\tan x} = \frac{0+0}{0+0+0+0}$$

$$\lim_{x\to 0} \frac{2\sec^2 x\tan x + \sin x}{2\tan x + 4x\sec^2 x + 2x^2\sec^2 x\tan x}$$

$$\lim_{x\to 0} \frac{4\sec x\cdot\sec x\tan x\cdot\tan x + 2\sec^2 x\cdot\sec^2 x + \cos x}{2\sec^2 x + 4\sec^2 x + 4x\cdot 2\sec x\cdot\sec x\tan x + [4x\sec^2 x + 2x^2\cdot 2\sec x\sec x\tan x]\tan x + 2x^2\sec^2 x\cdot\sec^2 x}$$

$$\lim_{x\to 0} \text{"} \quad \text{"} = \frac{0+2+1}{2+4+0+0+0} = \frac{3}{6} = \frac{1}{2}$$

5. $$\lim_{x\to \infty} \frac{x+7}{x^2+x+1} = \frac{\infty}{\infty}$$

$$\lim_{x\to \infty} \frac{1}{2x+1} = 0$$

6. $$\lim_{x\to \infty} \frac{x}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x\to \infty} \frac{1}{e^x} = 0$$

7. $\lim\limits_{x \to 3} \dfrac{x^2-9}{x^2-x-6} = \dfrac{0}{0}$

$\lim\limits_{x \to 3} \dfrac{2x}{2x-1} = \dfrac{6}{5}$

8. $\lim\limits_{x \to 0} \dfrac{2x - \sin x}{x} = \dfrac{0-0}{0}$

$\lim\limits_{x \to 0} \dfrac{2-\cos x}{1} = \dfrac{2-1}{1} = 1$

9. $\lim\limits_{x \to 0} \dfrac{x - \sin(2x)}{\tan x} = \dfrac{0-0}{0}$

$\lim\limits_{x \to 0} \dfrac{1-2\cos(2x)}{\sec^2 x} = \dfrac{1-2}{1} = -1$

10. $\lim\limits_{x \to -2} \dfrac{x^2+6x+8}{x^2-3x-10} = \dfrac{4-12+8}{4+6-10} = \dfrac{0}{0}$

$\lim\limits_{x \to -2} \dfrac{2x+6}{2x-3} = \dfrac{-4+6}{-4-3} = -\dfrac{2}{7}$

11. $\lim\limits_{x \to 2} \dfrac{x^2+x-6}{x-2} = \dfrac{4+2-6}{0} = \dfrac{0}{0}$

$\lim\limits_{x \to 2} \dfrac{2x+1}{1} = 5$

12. $\lim\limits_{x \to \infty} \dfrac{\ln(x^{10000})}{x} = \dfrac{\infty}{\infty}$

$\lim\limits_{x \to \infty} \dfrac{\dfrac{10000\,\cancel{x^{9999}}}{x^{\cancel{10000}\;1}}}{1} = 0$

13. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\ln(\sin x)^3}{\frac{\pi}{2} - x} = \dfrac{0}{0}$

$\lim\limits_{x \to \frac{\pi}{2}} \dfrac{3 \cdot \frac{\cos x}{\sin x}}{-1} = 0$

14. $\lim\limits_{t \to 1} \dfrac{\sqrt{t} - t^2}{\ln t} = \dfrac{1-1}{0} = \dfrac{0}{0}$

$\lim\limits_{t \to 1} \dfrac{\frac{1}{2}t^{-1/2} - 2t}{\frac{1}{t}} = \dfrac{\frac{1}{2} - 2}{1} = \dfrac{-3}{2}$

15. $\lim\limits_{x \to 0} \dfrac{\ln(\cos(2x))}{7x^2} = \dfrac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{\frac{-2\sin(2x)}{\cos 2x}}{14x} = \dfrac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{-2\tan(2x)}{14x}$

$\lim\limits_{x \to 0} \dfrac{-4\sec^2(2x)}{14} = \dfrac{-4}{14} = \dfrac{-2}{7}$

16. $\lim\limits_{x \to 0} \dfrac{\tan x - x}{\sin(2x) - 2x} = \dfrac{0-0}{0-0} = \dfrac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{\sec^2 x - 1}{2\cos(2x) - 2} = \dfrac{1-1}{2-2} = \dfrac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{2\sec x \cdot \sec x \tan x}{-4\sin(2x)} = \dfrac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{2\sec^2 x \tan x}{-4\sin(2x)}$

$\lim\limits_{x \to 0} \dfrac{4\sec x \cdot \sec x \tan x \cdot \tan x + 2\sec^2 x \cdot \sec^2 x}{-8\cos(2x)} = \dfrac{0+2}{-8} = -\dfrac{1}{4}$

17. $\lim\limits_{x \to \infty} \dfrac{\ln(\ln x^{1000})}{\ln x} = \dfrac{\infty}{\infty}$

$\lim\limits_{x \to \infty} \dfrac{\dfrac{\frac{1000x^{\cancel{999}}}{x^{\cancel{1000}}}}{\ln x^{1000}}}{\frac{1}{x}}$

$\lim\limits_{x \to \infty} \dfrac{1000}{x} \times \dfrac{1}{x \ln x^{1000}} = 0$

18. $\lim_{x \to 0} \dfrac{\tan^{-1} x - x}{8x^3} = \dfrac{0-0}{0} = \dfrac{0}{0}$

$\lim_{x \to 0} \dfrac{\frac{1}{1+x^2} - 1}{24x^2} = \dfrac{1-1}{0} = \dfrac{0}{0}$

$\lim_{x \to 0} \dfrac{(1+x^2)^{-1} - 1}{24x^2}$

$\lim_{x \to 0} \dfrac{-(1+x^2)^{-2}(2x)}{48x}$

$\lim_{x \to 0} \dfrac{\frac{-2x}{(1+x^2)^2}}{48x} = \dfrac{0}{0}$ | $\dfrac{-2x(1+x^2)^{-2}}{48x}$

$\lim_{x \to 0} \dfrac{-2(1+x^2)^{-2} + (-2x)(-2(1+x^2)^{-3} \cdot 2x)}{48} = \dfrac{-2+0}{48} = \dfrac{-1}{24}$

19. $\int e^{2x} \cos x \, dx$

$u = e^{2x}$ $\quad \int dv = \int \cos x \, dx$

$\frac{du}{dx} = 2e^{2x}$ $\quad v = \sin x$

$du = 2e^{2x} dx$

$\boxed{e^{2x} \sin x - 2} \int e^{2x} \sin x \, dx$

$u = e^{2x}$ $\quad \int dv = \int \sin x \, dx$

$\frac{du}{dx} = 2e^{2x}$ $\quad v = -\cos x$

$du = 2e^{2x} dx$

$-e^{2x} \cos x + 2\int e^{2x} \cos x \, dx$

$\int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x - 4\int e^{2x} \cos x \, dx$

$5\int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x + C$

$\int e^{2x} \cos x \, dx = \frac{1}{5}\left[e^{2x} \sin x + 2e^{2x} \cos x + C\right]$

20. $\int x^4 \cos(2x) dx$

	u	dv
+	x^4	$\cos(2x)$
−	$4x^3$	$\frac{1}{2}\sin(2x)$
+	$12x^2$	$-\frac{1}{4}\cos(2x)$
−	$24x$	$-\frac{1}{8}\sin(2x)$
+	24	$\frac{1}{16}\cos(2x)$
−	0	$\frac{1}{32}\sin(2x)$

$\frac{1}{2}x^4\sin(2x) + x^3\cos(2x) - \frac{3}{2}\sin(2x) - \frac{3}{2}\cos(2x) + \frac{3}{4}\sin(2x) + C$

21. $\int x^2 e^x dx$

$u = x^2$ $\quad\quad \int dv = \int e^x dx$

$\frac{du}{dx} = 2x$ $\quad\quad v = e^x$

$du = 2x\,dx$

$\boxed{x^2e^x - 2}\int xe^x dx$

$u = x$ $\quad\quad \int dv = \int e^x dx$

$\frac{du}{dx} = 1$ $\quad\quad v = e^x$

$du = dx$

$xe^x - \int e^x dx$

$x^2e^x - 2xe^x + 2e^x + C$

22. $\int \ln(3x) dx$

$u = \ln(3x)$ $\quad\quad \int dv = \int dx$

$\frac{du}{dx} = \frac{3}{3x}$ $\quad\quad v = x$

$du = \frac{1}{x} dx$

$x\ln(3x) - \int x \cdot \frac{1}{x} dx$

$x\ln(3x) - x + C$

23. $\int x\sec^2 x\,dx$

$u = x$ $\quad$ $\int dv = \int \sec^2 x\,dx$

$du = dx$ $\quad$ $v = \tan x$

$x\tan x - \int \tan x\,dx$

$x\tan x + \ln|\cos x| + C$

24. $\int x e^{-2x}\,dx$

$u = x$ $\quad$ $\int dv = \int e^{-2x}\,dx$

$du = dx$ $\quad$ $v = -\frac{1}{2}e^{-2x}$

$-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$

$-\frac{1}{2}xe^{-2x} + -\frac{1}{4}e^{-2x} + C$

25. $\int x\sin x\,dx$

$u = x$ $\quad$ $\int dv = \int \sin x\,dx$

$du = dx$ $\quad$ $v = -\cos x$

$-x\cos x + \int \cos x\,dx$

$-x\cos x + \sin x + C$